

Creation of entangled states in coupled quantum dots via adiabatic rapid passage

C. Creatore,¹ R. T. Brierley,¹ R. T. Phillips,¹ P. B. Littlewood,^{1,2,3} and P. R. Eastham⁴

¹*Cavendish Laboratory, University of Cambridge,
CB3 0HE, Cambridge, United Kingdom*

²*Argonne National Laboratory, Argonne IL 60439, USA*

³*James Franck Inst., University of Chicago, Chicago IL 60637*

⁴*School of Physics, Trinity College, Dublin 2, Ireland*

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Abstract

Quantum state preparation through external control is fundamental to established methods in quantum information processing and in studies of dynamics. In this respect, excitons in semiconductor quantum dots (QDs) are of particular interest since their coupling to light allows them to be driven into a specified state using the coherent interaction with a tuned optical field such as an external laser pulse. We propose a protocol, based on adiabatic rapid passage, for the creation of entangled states in an ensemble of pairwise coupled two-level systems, such as an ensemble of QD molecules. We show by quantitative analysis using realistic parameters for semiconductor QDs that this method is feasible where other approaches are unavailable. Furthermore, this scheme can be generically transferred to some other physical systems including circuit QED, nuclear and electron spins in solid-state environments, and photonic coupled cavities.

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Whether one is interested in methods of quantum optics or computation, or more general many-body dynamics, it is important to be able to prepare interacting few-level systems in initial states from pure product states to more complex entanglements. Excitons in low-dimensional semiconductors exemplify such a level structure. They are of interest both because their strong coupling to light provides communication with the external world, and because photon-mediated interactions between excitons can be substantial. However, the lack of degeneracy due to the (accidental or deliberate) inhomogeneous spread of the energy levels becomes a challenge for addressing different levels – either independently, for example to invert a single one, or in pairs, for entangling a specified set. Control of the population of excitonic states in quantum dots (QDs) has been demonstrated using resonant transform-limited light fields such as π Rabi pulses [1–4]. Although effective in certain cases, Rabi flopping has the disadvantage for population transfer that the final state is sensitive to variations in dipole couplings, dot energy and disorder. Such disorder becomes a serious challenge if one wishes to scale this approach beyond a single quantum dot, and similar limitations arise in many other systems. More recently, population transfer has been achieved [5, 6] using a protocol called adiabatic rapid passage (ARP) [7]. Within this scheme the frequency of the laser pulse is swept through that of the target state and, if whole process is performed adiabatically, this target state will be populated with high efficiency. Unlike the Rabi approach, ARP is largely unaffected by all the variations previously mentioned, and this has prompted theoretical proposals for its use in preparing populations in ensembles of QDs [8], as well as proposals to realize a non-equilibrium Bose-Einstein condensate [8–10] and quantum operations [11–17].

The aim of this paper is to show how ARP may be generalized to create entangled states in a disordered system, comprising a set of two-level systems coupled together in pairs. Such an ensemble could be realized using quantum dot molecules, with a possible intra-molecular coupling mechanism being resonant Förster transfer [13, 18–20]. Implementations would also be feasible in other systems such as photonic coupled cavities with photons tunneling between neighboring cavities and having onsite interaction [21], in arrays of superconducting qubits with an exchange coupling mediated by virtual photons [22, 23], or using electron spin states in coupled semiconductor impurities [24].

The important feature of our proposal is that it requires neither precise control, nor reproducibility, of the pairs. Instead we exploit the disorder to allow those pairs with

entangled states to be spectrally identified within an ensemble. The robustness of ARP then allows a pulse to be constructed which drives those pairs into their entangled states, without creating other excitations. Thus, as shown in Fig. 2, the entanglement of formation per excitation is very close to one, even in a strongly disordered system. Our approach represents a significant simplification and improvement of the protocol required for the production of entangled states in realistic systems, since previous proposals have relied on coupling two states through a further level [15], or consider only a single and fine-tuned system [11–13, 25, 26].

To simplify the notation we consider the limit in which the states of the individual quantum dots form two-level systems that may be represented in terms of a Bloch vector or pseudospin: each dot either contains no exciton (spin down) or a single exciton (spin up). The generalization to allow for multiple states on the dot, for example due to the exciton spin structure, is straightforward. The relevant Hamiltonian for two coupled dots, in a frame rotating with the time-dependent frequency $\omega(t)$ of the ARP pulse and in the rotating wave approximation, is then ($\hbar = 1$):

$$\hat{H} = \sum_{i=1,2} \left[\frac{\tilde{\varepsilon}_i}{2} \hat{\sigma}_i^z + g(t)(\hat{\sigma}_i^+ + \hat{\sigma}_i^-) \right] - j_T(\hat{\sigma}_1^+ \hat{\sigma}_2^- + \hat{\sigma}_2^- \hat{\sigma}_1^+), \quad (1)$$

where $\tilde{\varepsilon}_i = \varepsilon_i - \omega(t)$, ε_i is the transition energy for dot $i = 1, 2$, $g(t)$ the amplitude of the driving field used to perform ARP, σ_i the Pauli operators describing the state of dot i , and j_T an exchange interaction between the two sites ($\hbar = 1$). The ARP pulse can be decomposed into its amplitude and frequency, i.e., $F(t) = g(t)\exp(i \int \omega(t')dt')$. For definiteness we consider a Gaussian linearly chirped pulse:

$$F(t) = g(t)e^{i\omega(t)t} = g_0 e^{-t^2/2\tau^2} e^{i\omega(t)t}, \quad \omega(t) = \omega_0 + \alpha t, \quad (2)$$

with g_0 the pulse amplitude, τ the temporal width of the pulse, ω_0 its central frequency, and α the chirp. We take the pulse width to define units of time and energy, and use dimensionless parameters $\alpha\tau^2$, $g_0\tau$ and $j_T\tau$.

In general, ARP schemes are generalizations of the Landau-Zener problem [27, 28] to a time-dependent and controllable mixing $g(t)$ between the levels. Consider first noninteracting dots ($j_T = 0$): if the driving frequency sweeps through the transition frequency of a dot, $\tilde{\varepsilon}_i$ passes through zero, and the lowest energy eigenstate changes from an empty site to being fully occupied. In the Landau-Zener problem there is a constant hybridization g_0 between

these levels to create an anticrossing, and a system in the initial ground state will be driven into the excited state with probability $P_{\text{inv}} = 1 - \exp(-g_0^2/\alpha)$. Thus as the adiabaticity is increased, by reducing the chirp α or increasing the hybridization g_0 , the final population rapidly approaches one. In the case of ARP, the coupling term $g(t)$ is not constant and, in order for the transition to occur, the two levels of the system need to be coupled long enough so that the character of the eigenstates changes smoothly, i.e., adiabatically. This gives the criterion $\alpha\tau^2 \gg 1$ [7]. Furthermore, the ARP scheme can be made spectrally selective. One can choose to excite either one dot or the other by timing the center of the optical pulse to occur when the chirp has driven a particular level to zero, and ensuring that the pulse is negligible at the other crossing. The available discrimination in the energy levels is thus of order \hbar/τ .

This discrimination could be used to create entangled states in the ideal case of a pair of identical two-level systems [11–13]. In this case the Hamiltonian (1) is, in the absence of the driving field, diagonal in the singlet/triplet basis $\{|S\rangle, |T_-\rangle, |T_0\rangle, |T_+\rangle\}$, with $|S\rangle = (1/\sqrt{2})(|\downarrow\uparrow\rangle - |\uparrow\downarrow\rangle)$, $|T_-\rangle = |\downarrow\downarrow\rangle$, $|T_0\rangle = (1/\sqrt{2})(|\downarrow\uparrow\rangle + |\uparrow\downarrow\rangle)$, $|T_+\rangle = |\uparrow\uparrow\rangle$. The intermediate states $|T_0\rangle$ and $|S\rangle$ between the ground state and the fully occupied state $|T_+\rangle$ are spatially entangled if the coupling dominates over the detuning, $\Delta\varepsilon = \varepsilon_1 - \varepsilon_2 \lesssim j_T$. For a single pair of identical dots ($\varepsilon_1 = \varepsilon_2 = \varepsilon_0$), the use of ARP to create this entangled state is illustrated in Fig. 1(a) which shows the time evolution of the eigenvalues of the Hamiltonian Eq. (1) when the levels are coupled by a linearly chirped Gaussian pump ($g_0\tau = 5$, colored continuous lines) and uncoupled ($g_0\tau = 0$, thin dashed lines). By chirping through the level crossing (A) between the states $|T_-\rangle$ and $|T_0\rangle$ (at $\omega_0 = \varepsilon_0 - j_T$) with a pulse duration such that coupling switches off before the $|T_0\rangle \rightarrow |T_+\rangle$ crossing (B), the system will be adiabatically driven and left in the entangled triplet state $|T_0\rangle$.

We now consider the parameters required for this procedure for a single degenerate pair. In this case the population of the two-dot system in the desired final state is equally shared between $|\uparrow\downarrow\rangle$ and $|\downarrow\uparrow\rangle$, so $\rho_{\uparrow\downarrow} = \rho_{\downarrow\uparrow} = 0.5$. The element of the density matrix $\rho_{\uparrow\downarrow}$ ($= \rho_{\downarrow\uparrow}$), can thus be used as a measure of the insensitivity of the entanglement to variation of inter-dot coupling and chirp. This is illustrated in Fig. 1(b) which shows $\rho_{\uparrow\downarrow}$, obtained by numerical solution of the Liouville equation $d\rho/dt = -i[H, \rho]$, as a function of the dimensionless parameters $\alpha\tau^2$ and $j_T\tau$. The region where $\rho_{\uparrow\downarrow} = 0.5$ is seen to extend over a large range of values of chirp and exchange coupling. The required parameters are fully compatible with a

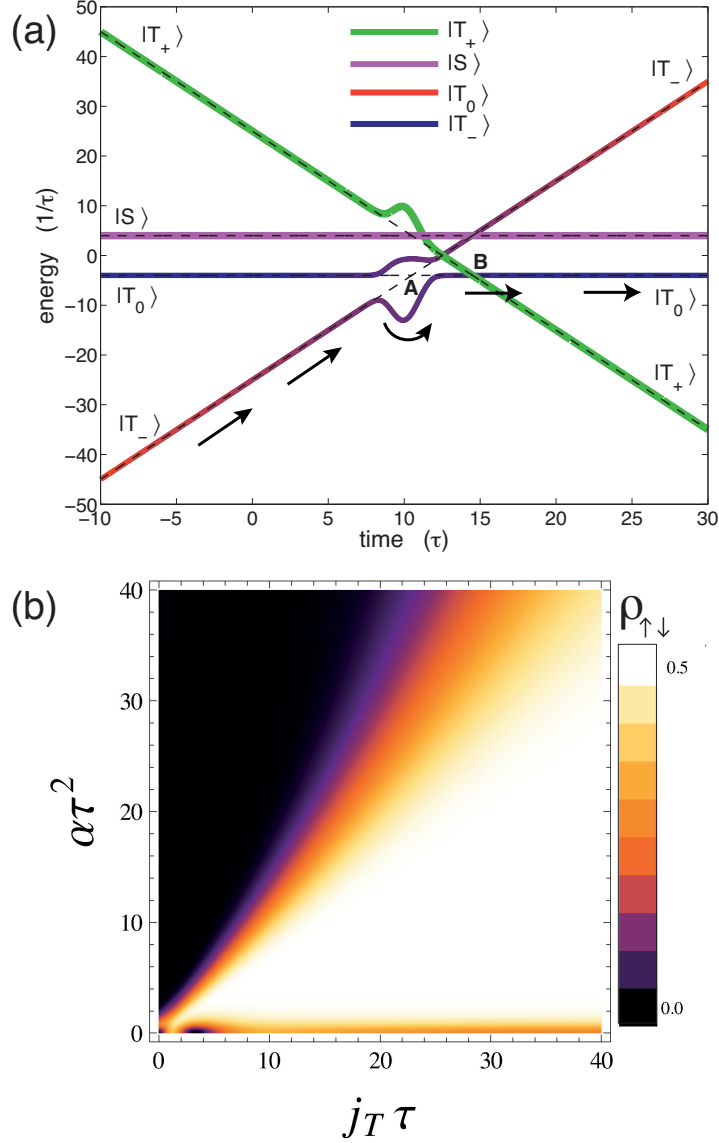


FIG. 1. (Color online) (a) Time-dependent energy levels of a coupled system of two identical dots. The continuous colored lines indicate the energy levels with a Gaussian driving pulse ($g_0\tau = 5$) of duration τ , while the thin dashed lines are the undriven levels ($g_0 = 0$). The chirp and interdot couplings are $\alpha\tau^2 = 2$ and $j_T\tau = 4$. The central frequency of the ARP pulse is $\varepsilon_0 - j_T$, resonant with the transition between the ground state $|T_- \rangle$ and the entangled state $|T_0 \rangle$ (see point A). (b) Final density matrix element $\rho_{\uparrow\downarrow}$, calculated as a function of the dimensionless linear chirp $\alpha\tau^2$ and exchange coupling $j_T\tau$. The white region ($\rho_{\uparrow\downarrow} = 0.5$) shows the range of values of chirp and exchange coupling where the pair is driven into the entangled state $|T_0 \rangle$ with high probability.

linearly chirped pulse similar to that used to invert a single semiconductor QD [5]: in that case we have considered a transform-limited pulse width of 2 ps and a chirped temporal width $\tau = 4.5$ ps. Thus for $\alpha\tau^2 \approx 2$ we require a dimensionless coupling $j_T\tau \gtrsim 4$ [see Fig. 1] for the ARP transition to the entangled state to occur, which corresponds to a value of Förster coupling of 0.6 meV. Previous studies have estimated an upper limit of 10 meV for Förster coupling in semiconductor quantum dots [20]. Thus the scheme could be implemented using two stacked (vertical) QDs at a distance of few nanometers, coupled by Förster energy transfer, but with no single-particle tunneling. Such conditions can be achieved in InAs/GaAs coupled QDs [29, 30]. The scheme would also apply with single-particle tunneling or indirect excitons, provided only that target entangled states can be identified.

We now show how this adiabatic protocol can be generalized to generate entanglement in realistic non-degenerate systems such as ensembles of QDs. In this case the entanglement pulse can be spectrally tuned to address a specific pair of dots, and several pairs can be entangled within an ensemble by superposing such pulses.

It is important to recognize that the requirement of exact degeneracy of the uncoupled transition is relaxed up to the magnitude of the coupling energy. This affords a route to practical realizations of the scheme, as the coupling energy and level splitting can be traded to optimize the probability of producing an appropriate double-dot structure. In the following we consider an ensemble of such systems, modeled here by an average coupling strength and having an inhomogeneous distribution of energies. In each coupled pair the energy levels are not degenerate, and only pairs with detunings smaller than the interdot coupling strength can be entangled; these specific pairs must be identified spectroscopically prior to ARP manipulation. Spectrally-selected components of a broad-band pulse, each close to resonance with a particular chosen pair, can then be chirped in the same linear optical process, such as a grating-based delay stage. We test this scheme simulating an ensemble consisting of thirty pairs of dots with energies taken from a Gaussian distribution of standard deviation 10 meV, and coupled by an average coupling strength $j_T = 2$ meV. In one typical realization taken as an example, three couples can be entangled as their energy levels are detuned by an amount smaller than $j_T/2$. To evaluate the entanglement produced, we use the *entanglement of formation* (EOF), which is a widely accepted measure of the

entanglement for bipartite states. For pure states it is the entropy [31]:

$$E(|\Psi\rangle) = S(\rho), \quad (3)$$

where $S(\rho) = -\text{Tr}(\rho^{\text{red}} \log_2 \rho^{\text{red}})$ is the von Neumann entropy and $\rho^{\text{red}} = \text{Tr}^{\text{red}} |\Psi\rangle\langle\Psi|$ is the reduced density matrix obtained by tracing the whole system density matrix $|\Psi\rangle\langle\Psi|$ over one of the two subsystems of which the pure state $|\Psi\rangle$ consists. In this case the two subsystems are the two paired dots. The EOF (or entropy) ranges from zero (for a product state) to $\log_2 N$ for a maximally entangled state of two N -state particles. Hence, the EOF of the triplet state $|T_0\rangle = (1/\sqrt{2})(|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle)$ (which is a maximally entangled Bell state) is equal to 1. Furthermore, the properties of the EOF as a natural measure of the entanglement include that the entanglement of independent systems, such as pairs of coupled dots in an ensemble, is additive.

Figure 2 shows the EOF and the total excitation in our example realization calculated as a function of the strength of the applied chirped Gaussian pulse. The excitation induced in the system after the application of an external pulse has been evaluated as $X = \text{Tr}(\rho\hat{X})$, the excitation operator \hat{X} given by $\hat{X} = (\sigma_1^z + 1)/2 + (\sigma_2^z + 1)/2$ and $\sigma_{1,2}^z$ being the two-particles spin operators for dot 1 and 2. We have used $\tau = 15$ ps and $\alpha = 0.01$ ps⁻² ($\alpha\tau^2=2.25$) yielding a small value of $\alpha\tau$ – which determines the energy range spanned by each component of the ARP pulse – while keeping the whole process in the adiabatic regime $\alpha\tau^2 > 1$, so that few states apart from the entangled ones will be excited. The total entanglement (dashed red curve) always deviates from the ideal value of 3, i.e., the total number of triplet states, due to the unavoidable excitation of other states which are not entangled, but yield a small amount of entropy. Furthermore, as the intensity of the pulse increases, other level crossings become adiabatic and other states such as the full inversion $|\uparrow\uparrow\rangle$ can be populated, thus explaining the behavior of the excitation (continuous black) curve. However, the EOF normalized with respect to the total excitation (dashed-dotted blue curve), which is an effective measure of the total entanglement produced in the system, is close to the ideal value of 1 (≈ 0.85 for all the different pump strengths considered). In order to validate these results we have simulated 50 different realizations of Gaussian-distributed coupled dots, and found an average value for the normalized EOF always ≈ 0.8 for several realistic values of pulse strengths.

In conclusion, we have shown the benefits of ARP for the generation of entangled states in ensembles of coupled quantum systems such as inhomogeneously distributed QD excitons.

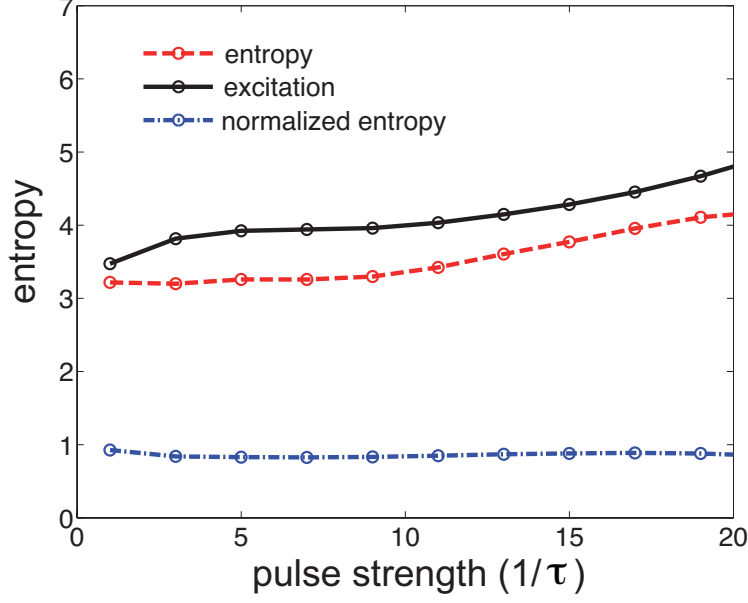


FIG. 2. (Color online) Predicted entanglement generated when an ensemble of 30 coupled pairs of dots is driven by an ARP pulse. The pulse is constructed to drive the 3 strongly-coupled pairs in the ensemble into their entangled state (see text). The red dashed curve shows the total entanglement entropy (or entropy of formation), the black solid curve the total excitation, and the blue dot-dashed curve the entanglement entropy normalized by the total excitation. The horizontal axis is the strength of the driving field, defined as twice the peak Rabi splitting in a single dot, given in units of $1/\tau$ ($\hbar = 1$).

Our calculations, based on realistic values for Förster coupling and dot distribution, provide a feasible route for the realization of entanglement in solid state systems, which is more practical than other approaches due to the flexible and tunable parameters of ARP. In principle, the same methodology can apply to microcavity Josephson junctions, to coupled electron spins in semiconductors (using ESR), and to coupled nuclear spins (via NMR). The latter arena is the genesis of the ARP technique, though we are not aware of it being explicitly used in the form proposed here.

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